

• Reviews •

A Survey of Repetitive Control for Nonlinear Systems

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Abstract *In aerospace engineering and industry, control tasks are often of a periodic nature, while repetitive control is especially suitable for tracking and rejection of periodic exogenous signals. Because of limited research effort on nonlinear systems, we give a survey of repetitive control for nonlinear systems in this paper. First, a brief introduction of repetitive control is presented. Then, after giving a brief overview of repetitive control for linear systems, this paper summarizes design methods and existing problems of repetitive control for nonlinear systems in detail. Lastly, relationships between repetitive control and other control schemes are analyzed to recognize repetitive control from different aspects more insightfully.*

Key words Repetitive control, nonlinear systems, survey

I. Introduction

In nature, numerous examples of periodic phenomena are found and observed, ranging from the orbital motion of the heavenly bodies to the rhythm of the heart. In aerospace engineering, many control tasks are often of a periodic nature as well. For example: magnetic spacecraft attitude control [1],[2], active control of vibrations in helicopters [3],[4], autonomous vertical landing on an oscillating platform [5],[6] and harmonics elimination in aircraft power supplies [7]. Besides these, in industrial manipulators executing operations of picking, placing or painting, machine tools and magnetic disk or CD drives, the control systems are often required to track or reject periodic exogenous signals. For these periodic control tasks, repetitive control (RC, or repetitive controller, also designated RC) often can achieve a

control performance with a high precision.

RC is an internal-model-based control approach in which the infinite-dimensional internal model $1/(1 - e^{-sT})$, gives rise to an infinite number of poles on the imaginary axis. The basic idea of RC is the cancelation viewpoint on the internal model principle (IMP) [8],[9]. RC was initially developed for continuous single-input, single-output linear time-invariant systems in [10], for high accuracy tracking of a periodic signal with a known period. Later, RC was extended to multiple-input multiple-output (MIMO) linear time-invariant (LTI) systems in [9]. Since then, RC has begun to receive more attention and applications, and has become a special topic in control theory. In recent years, the development on RC has been uneven. By the use of frequency methods, the theories and applications in LTI systems have developed very well. On the other hand, RC for nonlinear systems has received limited research effort. Taking these into account, we will give a survey of RC for nonlinear systems in this paper.

The rest of this paper is organized as follows. In Section II, a brief introduction of RC is presented. Then, a brief overview of RC for linear systems is given in Section III. In Section IV, design methods and existing problems of RC for nonlinear systems are summarized in detail. In Section V, relationships between RC and other control schemes are analyzed. Conclusions are given in Section VI.

II. Basic Idea of Repetitive Control

The basic idea of RC stems from the IMP. The IMP states that if any exogenous signal can be regarded as the output of an autonomous system, then the inclusion of this signal model, namely internal model, in a stable closed-loop system can as-

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sure asymptotic tracking and/or asymptotic rejection of the signal [8]. In order to achieve asymptotic tracking and/or asymptotic rejection, if a given signal is composed of a certain number of harmonics, then a corresponding number of neutrally stable internal models (one for each harmonic) should be incorporated into the closed loop according to the IMP. Integral control is a typical application. It is well known that integral control can track and reject any step external signal. This can be explained by the IMP as the models of an integrator and a step signal are the same. However, according to the IMP, it will bring in trouble to design a tracking controller for a general periodic signal. First, harmonics of a general periodic signal need to be analyzed. It is difficult or may cost much time to obtain accurate harmonics. Second, the controller will contain more neutrally stable internal models (one for each harmonic) as the number of harmonics increases. This will lead the controller structure too complex. Also, it will cost much time to solve these neutrally stable internal models (differential equations) to obtain the control output. The two drawbacks can be overcome by using RC. The innovation of RC is to construct the internal model for any T -periodic signal as follows [11]:

$$I_M \triangleq \frac{1}{1 - e^{-sT}} = \frac{1}{s \prod_{k=1}^{\infty} \left(1 + \frac{T^2 s^2}{4\pi^2 k^2}\right)}. \quad (1)$$

It can be observed that the internal model contains internal models of all sinusoidal functions with T period and the step signal. But, ironically, RC with the infinite number of harmonics has a simple structure! This confirms a Chinese proverb "things will develop in the opposite direction when they become extreme." A controller including the internal model (1) is called a RC and a system with such a controller is called a RC system [9].

In order to show IMP more explicitly, in the following, the IMP is used to explain the role of the internal models for step signals and T -periodic signals, respectively.

A. Step Signals

Since the Laplace transformation model of a unit step signal and an integrator are the same, namely $1/s$, the inclusion of the model $1/s$ in a stable closed-loop system can assure asymptotic tracking and/or asymptotic rejection of the unit step sig-

nal according to the IMP.

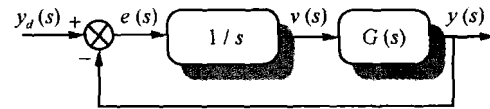


Fig. 1 Step signal tracking

As shown in Fig. 1, the transfer function from the desired signal to the tracking error is written as follows:

$$\begin{aligned} e(s) &= \frac{1}{1 + \frac{1}{s}G(s)} y_d(s) = \frac{1}{s + G(s)} \left(s \frac{1}{s}\right) \\ &= \frac{1}{s + G(s)}. \end{aligned}$$

Then, it only requires to verify whether or not the roots of the equation $s + G(s) = 0$ are all in the left s -plane, namely whether or not the closed-loop system is stable. If all roots are in the left s -plane, then the tracking error tends to zero as $t \rightarrow \infty$. Therefore, the tracking problem has been reduced to a stabilization problem of the closed-loop system.

B. T -periodic Signals

If the external signal is in the form of $y_d(t) = y_d(t - T)$, which can represent any periodic signal with a period T , then asymptotic tracking and/or asymptotic rejection can be achieved by incorporating the model $1/(1 - e^{-sT})$ into the closed-loop system.

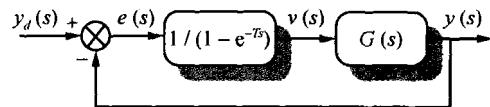


Fig. 2 T -periodic signal tracking

Similarly, the transfer function from the desired signal to the error is written as follows:

$$\begin{aligned} e(s) &= \frac{1}{1 + \frac{1}{1 - e^{-sT}}G(s)} y_d(s) \\ &= \frac{1}{1 - e^{-sT} + G(s)} \left[(1 - e^{-sT}) \frac{1}{1 - e^{-sT}}\right] \\ &= \frac{1}{1 - e^{-sT} + G(s)}. \end{aligned}$$

Then, it is only required to verify whether or not the roots of the equation $1 - e^{-sT} + G(s) = 0$ are all in the left s -plane. Therefore, the tracking problem has been reduced to a stabilization problem of the closed-loop system.

III. Brief Overview of RC for Linear Systems

RC is an internal-model-based control approach in which the infinite-dimensional internal model $1/(1-e^{-sT})$ gives rise to an infinite number of poles on the imaginary axis. It was proved in [9] that, for a class of general linear plants, exponential stability of RC systems could be achieved only when the plant is proper but not strictly proper. Moreover, the internal model $1/(1-e^{-sT})$ may destabilize the system. A linear RC system is a neutral type system in a critical case [12], [13]. Consider the following simple RC system:

$$\begin{aligned}\dot{x}(t) &= -x(t) + u(t) \\ u(t) &= u(t-T) - x(t)\end{aligned}$$

where $x(t), u(t) \in \mathbb{R}$. The RC system above can be also written to be a neutral type system in a critical case as follows:

$$\dot{x}(t) - \dot{x}(t-T) = -2x(t) + x(t-T).$$

The system above is in fact a neutral type system in a critical case [12], [13].

To enhance stability, a suitable filter is introduced as shown in Fig. 3, forming a *filtered repetitive controller* (FRC, or *filtered repetitive control*, also designated FRC) in which the loop gain is reduced at high frequencies¹ Stability results only with some sacrifice of high frequency performance. With appropriate design, however, an FRC can often achieve an acceptable tradeoff between tracking performance and stability, a tradeoff which broadens the application of RC in practice. The plug-in RC system shown in Fig. 3 is a widely used structure. Under the structure, the design objective is to design and optimize the filter $Q(s)$ and the compensator $B(s)$.

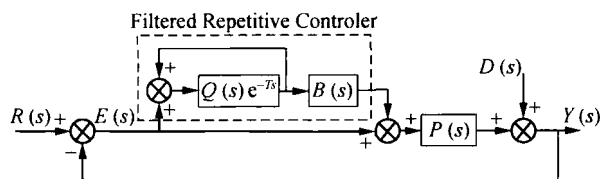


Fig. 3 Plug-in RC system diagram

After the development in past 30 years, a

great deal of research effort has been devoted to the theories and applications on RC for linear systems. About RC for linear systems, the interested readers could consult [14], [15], [16], [17] and references therein for the development. Currently, the research mainly focuses on robust RC [18], [19], [20], [21] and spatial-based RC [23]. Robust RC mainly includes two aspects: robustness to stability of closed-loopsystems [18], [19], [20] and robustness to uncertain or time-varying period-time [21], [22]. The researchers attempt to design better RCs to satisfy more and more practical various requirements.

IV. Repetitive Control for Nonlinear Systems

For nonlinear systems, it is not trivial to follow the idea of FRC because the related theories are derived in the frequency domain and can be applied only with difficulty, if at all, to nonlinear systems. Currently, there exist two major ways to design RCs for nonlinear systems.

A. Major Design Methods of RC

1) *Linearization Approach*: One way is to transform a nonlinear system into a linear system, then apply existing design methods to the transformed linear system. In the early years, researchers often only consider the following nonlinear system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + \phi(x, t) \\ y(t) &= Cx(t) + Du(t).\end{aligned}\quad (2)$$

This is related to the research of nonlinear systems in the early years. The RC design often had to restrict on the nonlinear term $\phi(x, t)$, such as Lipschitz conditions [24] or sector conditions [25]—[26]. Along with the appearance of feedback linearization and backstepping, the RC design of nonlinear systems develops further. By these new techniques, some nonlinear systems can be transformed into the form of (2) with some restrictions. Some existing design methods can be used directly.

Along with the appearance of feedback linearization and backstepping, the RC design for nonlinear systems develops further [27]—[31]. Differential

¹ In this note we have replaced the term “modified” in [9] with the more descriptive term “filtered”.

geometric techniques are combined with the IMP resulting in a nonlinear RC strategy. A formulation is presented for the case of input-state linearizable and input-output linearizable systems in continuous time [27]. By the input-output linearized method and the approximate input-output linearized method, the applicability of the finite-dimensional RC to nonlinear tracking control problems is studied for three different classes of nonlinear systems: 1) with a well-defined relative degree, 2) which fail to have a well-defined relative degree, and 3) linear plants with small actuator nonlinearity [28]. By using feedback linearization and output redefinition, a RC is developed to achieve precise periodic signal tracking control of single-input single-output nonlinear non-minimum phase systems [29], [30]. By backstepping, RC design and analysis are developed for backstepping controlled nonlinear systems [31]. The backstepping control employs both feedback and feedforward actions to render linearized I/O plant and thus the outer loop RC design can be based on the compensated linear system.

Problem: By these new techniques, some nonlinear systems can be transformed into linear systems subject to nonlinear terms more easily. Existing design methods can be used directly, which facilitates the RC design. However, not all nonlinear systems can be transformed into the familiar form, or the resulting nonlinear terms are difficult to handle.

2) *Adaptive-control-like Approach*: The other major way is to convert a tracking problem of nonlinear systems into a rejection problem of nonlinear error dynamics, then apply existing adaptive-control-like approach to the converted error dynamics. Concretely, there exist two design methods, namely Lyapunov-based (LB) approach [32]—[37] and evaluation-function-based approach [41]—[48]. The former is only applicable to RC design, but the latter is applicable to both RC design and iterative learning controller (ILC, or iterative learning control, also designated ILC) design. To clarify the previous work, we assume that v is a learning variable, v_d is a desired signal, and $\bar{v} = v_d - v$ is the learning error.

The Lyapunov-based approach is similar to the traditional adaptive control approach, where v_d is a T -periodic signal for the former and is a constant

for the latter. So, the resulting controllers are also called as ‘adaptive’ repetitive learning controllers [36]. To adopt the traditional adaptive control (AC) approach or the LB approach, we need to obtain the nonlinear error dynamics in the form of

$$\dot{e}(t) = f(e, t) + b(e, t)\bar{v}(t), \quad (3)$$

where e is the error. Because of the different desired signals, the chosen Lyapunov functions and the designed controllers are different, shown in Table 1.

Table 1 The Differences

	Lyapunov function	Controller
AC	$\bar{v}^T(t)\bar{v}(t)$	$\dot{v}(t) = h(x, t)$
LB	$\int_{t-T}^t \bar{v}^T(\theta)\bar{v}(\theta)d\theta$	$v(t) = v(t-T) + h(x, t)$

Currently, the AC approach is the leading method of designing RCs in nonlinear systems. This approach is first applied to the control of robot manipulators [32]. Subsequently, an intermediate result (an assumption) is given in [33] to form the framework of the LB approach.

Intermediate Result: The functions $f: [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $b: [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ are bounded when $e(t)$ is bounded on $\mathbb{R}^+$. Moreover, there exists a differentiable function $V: [0, \infty) \times \mathbb{R}^n \rightarrow [0, \infty)$, a positive definite matrix $M(t) = M^T(t) \in \mathbb{R}^{n \times n}$ with $0 < \lambda_M I_n < M(t)$ and a matrix $F(t, e(t)) \in \mathbb{R}^{n \times m}$ such that

$$\dot{V}(t, e(t)) \leq -e^T(t)M(t)e(t) + F^T(t, e(t))\bar{v}(t). \quad (4)$$

Based on the intermediate result, the controller $v(t) = v(t-T) + F(t, e(t))$ can ensure the tracking error approaching zero. The proof needs to employ a Lyapunov functional $V(t, e(t)) + 1/2 \int_{t-T}^t \bar{v}^T(s)\bar{v}(s)ds$ and Barbalat’s Lemma.

A novel learning approach is described in [34] for asymptotic state tracking in a class of nonlinear systems. Compared with the previous methods, the best advantage of the proposed learning approach is computationally simple and does not require one to solve any complicated equations based on full system dynamics. Hybrid control schemes are developed, which utilize learning-based feedforward terms to compensate for periodic dynamics and other Lyapunov-based approaches to compensate for aperiodic dynamics [35]. A Lyapunov

based adaptive RC is proposed for a class of nonlinear parameterized systems [36]. Both partially and fully saturated learning laws are analyzed in detail, and compared analytically. By considering that many RC schemes require the plant to be parameterizable, a repetitive learning is integrated with adaptive robust control by using the backstepping design for a class of cascade systems without parameterizations in [37]. A continuous universal repetitive learning control is proposed in [38] to track periodic trajectory for a class of nonlinear dynamical systems with nonparametric uncertainty and unknown state-dependent control direction matrix. In order to achieve a tradeoff between tracking performance and stability, an FRC is proposed for a class of nonlinear systems [39]. More importantly, the proposed FRC can deal with small input delay while the corresponding RC cannot.

The evaluation-function-based approach is applicable to design both ILCs [40] and RCs. The evaluation function is often formulated as follows [41]–[48]:

$$E_i = \int_0^T \bar{v}_i^T(\theta) \bar{v}_i(\theta) d\theta$$

where $\bar{v}_i(\theta) \triangleq \bar{v}(iT + \theta)$, $\theta \in [0, T]$, T is the period and $i = 0, 1, 2, \dots$ is the iterative number. The objective is often to design ILCs for the resetting condition and RCs for the alignment condition to result in the relationship as follows:

$$\Delta E_i = E_i - E_{i+1} \leq \alpha (\|e_i(0)\|^2 - \|e_i(T)\|^2) - \beta \int_0^T \|e_i(\theta)\|^2 d\theta$$

where $e_i(\theta) \triangleq e(iT + \theta)$, $\theta \in [0, T]$, $\alpha, \beta > 0$. Under the resetting condition, we have $\|e_i(0)\| = 0$. While, under the alignment condition, we have $\|e_{i+1}(0)\| = \|e_i(T)\|$. No matter under which condition, we can obtain

$$\beta \lim_{i \rightarrow \infty} \sum_{k=0}^i \int_0^T \|e_k(\theta)\|^2 d\theta \leq E_0 + \alpha \|e_0(0)\|.$$

Sequentially, by Barbalat's Lemma, it can be proved that the tracking error approaches zero as $t \rightarrow \infty$.

In the early years, the researchers mainly consider the resetting condition by using the idea above. The the alignment condition is analyzed in [14]. This work has greatly stimulated the more recent development of both ILCs and RCs. [14], the backstepping technique is combined together

with the learning control mechanism for developing a constructive control strategy to cope with nonlinear systems subject to both structured periodic and unstructured aperiodic uncertainties. RC schemes based upon the use of a proportional-derivative (PD) feedback structure is proposed in [45], for which an iterative term is added to cope with the unknown parameters and disturbances. The proposed adaptive ILC of robot manipulators is further improved in [47]. Fully saturated adaptive RC for trajectory tracking of uncertain robotic manipulators is presented in [48].

Problem: Currently, the adaptive-control-like approach is the leading method of designing RCs in nonlinear systems. The structures of RCs obtained for the linear and nonlinear systems are similar or the same, but the ways to obtain these controllers are very different. By recalling Section II, the tracking problem can be reduced to a stabilization problem of the closed-loop system. So, we do not need to obtain error dynamics for LTI systems. However, for nonlinear systems, it is often required to derive error dynamics to convert a tracking problem to a disturbance rejection problem as (3). This in fact follows the idea of general tracking controller design, whereas the special feature of periodic signals is under-exploited. Therefore, the general tracking controller design will not only restrict the development of RC, but also fail to represent the special feature and importance of RC. For nonlinear non-minimum phase systems, the ideal internal dynamics are required to derive the error dynamics. This is difficult and computationally expensive especially when the internal dynamics are subject to an unknown disturbance [49]. As a result, the authors suppose that this is the reason why few RC works on such systems have been reported.

3) *Others:* A formalism of ILC is used in [50] to solve a RC problem of forcing a system to track a prescribed periodic reference signal. The proposed method adopts the idea of contraction mapping. However, the proposed method is only applicable to discrete-time systems. Moreover, it cannot be applied to rejection of periodic disturbances. A Quasi-Sliding Mode (QSM) based tracking control method is proposed for tackling MIMO nonlinear continuous-time systems with un-matching system uncertainties and exogenous disturb-

ances [51]. The QSM-based RC is with the prominent characteristics of invariance and robustness to parameter variations and exogenous disturbances. However, the proposed controller needs the derivative of state, which is often difficult to obtain accurately in practice.

B. Existing Problems of RC

A linear RC system is a neutral type system in a critical case [12], [13]. The characteristic equation of the neutral type system has an infinite sequence of roots with negative real parts approaching zero, i. e. $\sup\{\text{Re}s|F(s)=0\}=0$, where $F(s)$ is the characteristic equation. This implies that a sufficiently small uncertainty may lead to $\sup\{\text{Re}s|F(s)=0\}>0$. It is proved in [39] that a linear RC system will lose its stability when subject to an input delay no matter how small the delay is. Therefore, the stability of RC systems is insufficiently robust. The simulations in [39] further show that a nonlinear RC systems will lose its stability when subject to a small input delay as well. In practice, input delay is very common. Therefore, it is important to design a RC to deal with small input delay. Besides input delay, the following problems also need to be considered.

1) In linear systems, most of literature pays attention to design digital RC. However, the research on RC for nonlinear systems is almost a blank. Currently, controllers are generally realized by digital computers. With insufficient robustness of RC systems in mind, it is still unknown if the discretized controllers destabilize the original systems. Therefore, it is important to establish theories on designing digital RCs for nonlinear systems.

2) Most of RC designs require the period known a prior. In practice, the period cannot be known exactly. How to design a RC to cope with uncertain period is very practical as well. For linear systems, quite a few researchers improve RCs to deal with uncertain period [21]. However, there exists few research on nonlinear RC systems subject to uncertain period.

RC is a specific tracking control. So, besides the problems above, how to design RC for nonlinear non-minimum phase systems and underactuated nonlinear systems, etc, are still interesting.

As shown in Fig. 4, the periodic signal tracking problem is an instance of the general signal tracking problem, and in turn includes the stability

problem (means zero signal tracking problem here) as a special case. Consequently, periodic signal tracking should certainly be easier than general signal tracking. Nevertheless, if the RCs are designed by following existing methods used for general signal tracking problem, then the special feature of periodic signals is in fact under-exploited. Therefore, general methods will not only restrict the development of RC, but also fail to represent the special feature and importance of RC. Since periodic signals are special, it is believed that there should exist other methods, different from the general methods, to design RCs for nonlinear systems. With this in mind, a new viewpoint on the IMP and then a new design method are proposed in [52]. Furthermore, the proposed design method is applied to periodic signal tracking of nonlinear non-minimum phase systems. However, the theories therein need to be improved and completed further.

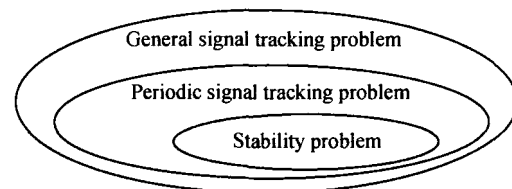


Fig. 4 Relationship between stability and tracking

V. Relation between RC and Other Control Methods

By taking RC as a class of adaptive control, ‘adaptive’ repetitive learning control design develops well these years. However, as analysis above, each design exists some drawbacks itself. So, it encourages us to recognize RC from different points of view. In the following, we will analyze relationships between RC and other control schemes.

(1) PID control and RC

Suppose a RC to be

$$u(t) = u(t-T) + T(k_1 e(t-T) + k_p \dot{e}(t-T) + k_d \ddot{e}(t-T)).$$

Then

$$\frac{u(t) - u(t-T)}{T} = k_1 e(t-T) + k_p \dot{e}(t-T) + k_d \ddot{e}(t-T).$$

If $T \rightarrow 0$ and the limits of both sides above exist, then

$$\begin{aligned}\dot{u}(t) &= \lim_{T \rightarrow 0} \frac{u(t) - u(t-T)}{T} \\ &= \lim_{T \rightarrow 0} (k_I e(t-T) + k_P \dot{e}(t-T) + k_D \ddot{e}(t-T)) \\ &= k_I e(t) + k_P \dot{e}(t) + k_D \ddot{e}(t).\end{aligned}$$

Furthermore, the equation above can be written as

$$u(t) = k_I \int_0^t e(s) ds + k_P e(t) + k_D \dot{e}(t) + c$$

where c is a constant. When $c=0$, the RC becomes a PID controller. In other words, PID control is a special case of RC as $T \rightarrow 0$. It is well known that PID control has been modified and extended greatly. Along these ideas, it is interesting to consider how to apply the existing modified and extended ways to RC.

(2) Optimization and RC

In order to express more explicitly, a simple example is given as follows: the following dynamic system

$$\dot{e}(t) = f(e, u, d, t)$$

is subject to disturbance d with a period T . Suppose that the following RC

$$u(t) = u(t-T) + e(t-T) \quad (5)$$

is adopted to make the tracking error $e(t) \rightarrow 0$ as $t \rightarrow \infty$.

Let us restate the control problem and design from optimization. Suppose $u(t), e(t), d(t)$ to be continuous and define $s_k(\theta) \triangleq s((k-1)T + \theta)$, $\theta \in [0, T]$, $s = (e, u, d)$, $k = 1, 2, \dots$. Then $s_k \in C[0, T]$ and $s_{k-1}(T) = s_k(0)$, where $C[0, T]$ is the space of continuous functions mapping $[0, T]$ into $\mathbb{R}$. Under the definitions above, the controller (5) can be rewritten as

$$u_k = u_{k-1} + e_{k-1}.$$

Since disturbance d is periodic, $d_k = d_{k-1}$, and then the dynamic system can be rewritten $\dot{e}_k = F(e_{k-1}, u_k)$. In space $C[0, T]$, we define an inner product as follows:

$$\langle x, y \rangle = \int_0^T x(\theta) y(\theta) d\theta, \quad x, y \in C[0, T].$$

By the definition, the optimization objective can be rewritten as $\langle e_k, e_k \rangle$. Then the RC can be rewritten as

$$\begin{aligned}\min & \langle e_k, e_k \rangle \\ \text{s. t. } & \dot{e}_k = F(e_{k-1}, u_k).\end{aligned}$$

Here iterative algorithm is $u_k = u_{k-1} + e_{k-1}$. In sense of optimization, the optimization problem can be also modified to be

$$\begin{aligned}\min & w_1 \langle u_k, u_k \rangle + w_2 \langle e_k, e_k \rangle \\ \text{s. t. } & \dot{e}_k = F(e_{k-1}, u_k).\end{aligned}$$

where w_1 and w_2 are weights.

(3) Dynamic feedback control and RC

The designers often want to avoid the time delay. In contrast, the time delay is brought into RCs. An FRC is in fact a dynamics feedback controller. A simple form of FRC is shown as follows [53]:

$$\begin{aligned}\dot{x}_c(t) &= -w_c x_c(t) + w_c x_c(t-T) + e(t) \\ u(t) &= w_c x_c(t-T) + e(t)\end{aligned}$$

where x_c is an auxiliary variable.

(4) Intelligent control and RC

A Russian proverb is "repetition is the mother of learning". In the time domain, a RC has represented a type of learning, where the current output of RC has used the last output and tracking error. Although the learning of RC is a low-level intelligence, it can be described by mathematic language and convergence can be proved. What does RC illuminate us? When human learns how to perform a periodic task, his attention is often focused on particular points in the task; for example, in downhill skiing, sharp turns are surely remembered and carefully negotiated. For this reason, a new concept: segmented RC is proposed in [16].

All the analysis above can help us recognize RC more insightful. In the future work, we can strengthen the relation between RC and other control methods. This may help us to develop more new 'derived' type of RC to satisfy different requirements.

V. Conclusions

In recent years, the development on RC has been uneven. The theories and applications in LTI systems have developed very well. On the other hand, RC for nonlinear systems has received limited research effort. Taking this into account, we give a survey of RC for nonlinear systems in this paper. Currently, there exist two major ways to design RCs for nonlinear systems: 1) linearization approach and 2) adaptive-control-like approach. Each of them exists some problems. It is expected

to develop more new design methods. In order to recognize RC more insightful and develop more design methods, relationships between RC and some control schemes are given.

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